

PHILOSOPHICAL TRANSACTIONS.

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The Squaring of the Hyperbola, by an infinite series of Rational Numbers, together with its Demonstration, by that Eminent Mathematician, the Right Honourable the Lord Viscount Brouncker.

WHat the Acute Dr. John Wallis had intimated, some years since, in the Dedication of his Answer to M. Meibomius de proportionibus, vid. That the World one day would learn from the Noble Lord Brouncker, the *Quadrature* of the *Hyperbole*; the Ingenious Reader may see performed in the subjoyned operation, which its Excellent Author w's now pleas'd to communicate, as followeth in his own words;

My Method for Squaring the Hyperbola is this :

Let AB be one *Asymptote* of the Hyperbola Ed C; and let AE and BC be parallel to throther: Let also AE be to BC as 2 to 1; and let the Parallelogram ABDE equal 1. See Fig. 1. And note, that the Letter x every where stands for Multiplication.

Supposing the Reader knows, that EA. $\alpha\beta$. KH. $\beta\gamma$. d δ . $\gamma\kappa$. $\delta\lambda$. $\epsilon\mu$. CB.&c. are in an *Harmonic series*, or a *series reciproca primanorum seu arithmetice proportionalium* (otherwise he is referr'd for satisfaction to the 87, 88, 89, 90, 91, 92, 93, 94, 95, prop. *Arithm. Infinitor. Wallisij* :)

$$\left. \begin{aligned} \text{I say } ABCdEA &= \frac{1}{1 \times 2} + \frac{1}{3 \times 4} + \frac{1}{5 \times 6} + \frac{1}{7 \times 8} + \frac{1}{9 \times 10} \&c. \\ EdCDE &= \frac{1}{2 \times 3} + \frac{1}{4 \times 5} + \frac{1}{6 \times 7} + \frac{1}{8 \times 9} + \frac{1}{10 \times 11} \&c. \\ EdCyE &= \frac{1}{2 \times 3 \times 4} + \frac{1}{4 \times 5 \times 6} + \frac{1}{6 \times 7 \times 8} + \frac{1}{8 \times 9 \times 10} \&c. \end{aligned} \right\} \text{in infinitum.}$$

For (in Fig. 2, & 3) the Parallelog.

And (in Fig. 4.) the Triangl.

CA = $\frac{1}{1 \times 2}$	EdC = $\frac{1}{2 \times 3 \times 4} = \frac{\square dD - \square dF}{2}$	<p><i>Note.</i></p> <p>CA = dD + dF</p> <p>dD = br + bn</p> <p>dF = fG + fk</p> <p>br = aq + ap</p> <p>bn = cs + cm</p> <p>fG = et + el</p> <p>fk = gu + gh</p> <p>&c.</p>
dD = $\frac{1}{2 \times 3}$ dF = $\frac{1}{3 \times 4}$	Ebd = $\frac{1}{4 \times 5 \times 6} = \frac{\square br - \square bn}{2}$	
br = $\frac{1}{4 \times 5}$ bn = $\frac{1}{5 \times 6}$	d fC = $\frac{1}{6 \times 7 \times 8} = \frac{\square fG - \square fk}{2}$	
fG = $\frac{1}{6 \times 7}$ fk = $\frac{1}{7 \times 8}$	Eab = $\frac{1}{8 \times 9 \times 10} = \frac{\square aq - \square ap}{2}$	
aq = $\frac{1}{8 \times 9}$ ap = $\frac{1}{9 \times 10}$	bcd = $\frac{1}{10 \times 11 \times 12} = \frac{\square cs - \square cm}{2}$	
cs = $\frac{1}{10 \times 11}$ cm = $\frac{1}{11 \times 12}$	def = $\frac{1}{12 \times 13 \times 14} = \frac{\square et - \square el}{2}$	
et = $\frac{1}{12 \times 13}$ el = $\frac{1}{13 \times 14}$	fgC = $\frac{1}{14 \times 15 \times 16} = \frac{\square gu - \square gh}{2}$	
gu = $\frac{1}{14 \times 15}$ gh = $\frac{1}{15 \times 16}$	&c.	
&c.	&c.	

And

And that therefore in the first series half the first term is greater than the sum of the two next, and half this sum of the second and third greater than the sum of the four next, and half the sum of those four greater than the sum of the next eight, &c. in infinitum. For $\frac{1}{2} dD = br + bn$; but $bn > fG$, therefore $\frac{1}{2} dD > br + fG$, &c. And in the second series half the first term is less than the sum of the two next, and half this sum less than the sum of the four next, &c. in infinitum.

That the first series are the even terms, viz. the 2^d, 4th, 6th, 8th, 10th, &c. and the second, the odd, viz. the 1st, 3^d, 5th, 7th, 9th, &c. of the following series, viz. $\frac{1}{1 \times 2} \cdot \frac{1}{2 \times 3} \cdot \frac{1}{3 \times 4} \cdot \frac{1}{4 \times 5} \cdot \frac{1}{5 \times 6} \cdot \frac{1}{6 \times 7}$, &c. in infinitum = 1. Whereof a being put for the number of terms taken at pleasure, $\frac{1}{a-1} \cdot \frac{1}{a}$ is the last, $\frac{a}{a+1}$ is the sum of all those terms from the beginning, and $\frac{1}{a+1}$ the sum of the rest to the end.

That $\frac{1}{4}$ of the first terme in the third series is less than the sum of the two next, and a quarter of this sum, less than the sum of the four next, and one fourth of this last sum less than the next eight, I thus demonstrate.

Let a = the 3^d or last number of any term of the first Column, viz. of Divisors,

$$\frac{1}{a} \cdot \frac{1}{a-1} \cdot \frac{1}{a-2} = \frac{1}{a^3 - 3a^2 + 2a} = \frac{16a^3 - 48a^2 + 56a - 24}{16a^6 - 96a^5 + 232a^4 - 288a^3 + 184a^2 - 48a} = A$$

$$\left. \begin{aligned} \frac{1}{2a} \cdot \frac{1}{2a-1} \cdot \frac{1}{2a-2} &= \frac{1}{8a^3 - 12a^2 + 4a} \\ \frac{1}{2a-2} \cdot \frac{1}{2a-3} \cdot \frac{1}{2a-4} &= \frac{1}{8a^3 - 36a^2 + 52a - 24} \end{aligned} \right\} = \frac{16a^3 - 48a^2 + 56a - 24}{64a^6 - 384a^5 + 880a^4 - 960a^3 + 496a^2 - 96a} = B$$

$$\frac{64a^6 - 384a^5 + 928a^4 - 1152a^3 + 736a^2 - 192a}{64a^6 - 384a^5 + 880a^4 - 960a^3 + 496a^2 - 96a} \times \frac{1}{4} A < B.$$

And $48a^4 - 192a^3 + 240a^2 - 96a$ = Excess of the Numerator above Denominator.

$$\begin{array}{l} \text{But --- The affirm.} > \text{the Negat.} \\ \text{That is, } 48a^4 + 240a^2 > 192a^3 + 96a \\ \text{Because } \begin{array}{l} a^4 + 5a^2 \\ a^4 + 5a \end{array} > \begin{array}{l} 4a^3 + 2a \\ 4a^3 + 2 \end{array} \end{array} \left. \vphantom{\begin{array}{l} a^4 + 5a^2 \\ a^4 + 5a \end{array}} \right\} \text{if } a > 2.$$

Therefore $B > \frac{1}{4} A$.

Therefore $\frac{1}{4}$ of any number of A; or Terms, is less than their so many respective B. that is, than twice so many of the next Terms. *Quod, &c.*

By

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By any one of which three Series, it is not hard to calculate, as near as you please, these and the like *Hyperbolic* spaces, whatever be the *Rational* Proportion of *A E* to *B C*. As for Example, when *A E* is to *B C*, as 5 to 4. (whereof the Calculation follows after that where the Proportion is, as 2 to 1. and both by the third Series.)

First then when (in Fig. 1.) $A E. B C :: 2. 1.$

2 x 3 x 4	I. (0.0416666666—)	0.0416666666	
4 x 5 x 6	I. (0.0083333333—)	0.0113095237	
6 x 7 x 8	I. (0.0029761904—)		
8 x 9 x 10	I. (0.0013888888—)		
10 x 11 x 12	I. (0.000575757—)	0.0029019589	
12 x 13 x 14	I. (0.0004578754—)		
14 x 15 x 16	I. (0.0002976190—)		
16 x 17 x 18	I. (0.0002042484—)		
18 x 19 x 20	I. (0.0001461988—)		
20 x 21 x 22	I. (0.0001082251—)		
22 x 23 x 24	I. (0.0000823452—)	0.0007306482	
24 x 25 x 26	I. (0.0000641026—)		
26 x 27 x 28	I. (0.0000508751—)		
28 x 29 x 30	I. (0.0000410509—)		
30 x 31 x 32	I. (0.0000336021—)		
32 x 33 x 34	I. (0.0000278520—)	0.0416666666	
34 x 35 x 36	I. (0.0000233406—)	0.0113095237	
36 x 37 x 38	I. (0.0000197566—)	0.0029019589	
38 x 39 x 40	I. (0.0000168691—)	0.0007306482	
40 x 41 x 42	I. (0.0000145180—)	3) 0.0001829939 (0.0000609980	
42 x 43 x 44	I. (0.0000125843—)	0.05679179	
44 x 45 x 46	I. (0.0000109793—)	+ 0.00006100	
46 x 47 x 48	I. (0.0000096361—)	0.05685279 < Ed Cy	
48 x 49 x 50	I. (0.0000085034—)		
50 x 51 x 52	I. (0.0000075415—)		
52 x 53 x 54	I. (0.0000067193—)	But 0.0007306482	
54 x 55 x 56	I. (0.0000060125—)	0.0001829939	
56 x 57 x 58	I. (0.0000054014—)	0.0000458315	
58 x 59 x 60	I. (0.0000048704—)		
60 x 61 x 62	I. (0.0000044068—)		
62 x 63 x 64	I. (0.0000040002—)		

Therefore 0.05679179
+ 0.00004583
+ 0.00001528
0.05685290 > Ed Cy.

For, it has been demonstrated that; of any terme in the last Column is less than the terme next after it; and therefore that; of the last terme, at which you stop

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top, is less than the remaining terms, and that the total of these is less than $\frac{1}{3}$ of a third proportional to the two last.

$$\begin{aligned} \text{And therefore } ABCyE \text{ being} &= 0.75 \text{ --- --- --- } 0.75 \\ &\text{and } EdCy > 0.05685279 \text{ --- --- --- } \text{and} < 0.05685290 \\ \text{And } ABCdE \text{ is} &< 0.69314720 \text{ --- --- --- } \text{and} > 0.69314709 \end{aligned}$$

But when $AE : BC :: 5 : 4$. or as EA . to KH . then will the space $ABCE$. or now, the space $AHKE$ ($AH = \frac{1}{2}AB$.) be found as follows.

$$\begin{array}{rcl} 8 \times 9 \times 10 & 1 & (0.0013888888 \\ 16 \times 17 \times 18 & 1 & (0.0002042484 \\ 18 \times 19 \times 20 & 1 & (0.0001461988 \\ 32 \times 33 \times 34 & 1 & (0.0000278520 \\ 34 \times 35 \times 36 & 1 & (0.0000233426 \\ 36 \times 37 \times 38 & 1 & (0.0000197566 \\ 38 \times 39 \times 40 & 1 & (0.0000168697 \end{array} \left. \begin{array}{l} \\ \\ \\ \\ \\ \\ \end{array} \right\} \begin{array}{l} 0.0003504472 \\ 0.0000878204 \\ 0.0000220074 \\ 0.0000073358 \\ 0.00000220074 \\ 0.00000073358 \\ 0.000000220074 \end{array} \begin{array}{l} 0.003888888 \\ 0.0003504472 \\ 0.0000878204 \\ 0.0000220074 \\ 0.0000073358 \\ 0.00000220074 \\ 0.00000073358 \end{array} \begin{array}{l} 3) 0.0000878204 (0.0000292735 \\ 0.001871564 \\ 0.0000292735 \\ 0.0018564299 < Eab \\ \text{But } 0.0003504472 \\ 0.0000878204 \\ 0.0000220074 \end{array} \begin{array}{l} \\ \\ \\ \\ \\ \\ \end{array} \left. \begin{array}{l} \\ \\ \\ \\ \\ \\ \end{array} \right\} \begin{array}{l} 0.0018271564 \\ + 0.0000220074 \\ + 0.0000073358 \\ 0.0018564996 > Eab \end{array}$$

Therefore EMb . (Fig 4.)

$$\begin{aligned} \text{being} &= 0.025 \text{ --- --- --- } 0.025 \\ Eab &> 0.0018564299 \text{ --- --- --- } &< 0.0018564996 \end{aligned}$$

$$\begin{aligned} EMba(\text{Fig. 4.}) \text{ or } EKM(\text{Fig. 1.}) &> 0.02685643 \text{ --- --- --- } < 0.02685650 \\ AHKM &< 0.22314356 \text{ --- --- --- } > 0.22314349 \end{aligned}$$

Therefore $3 ABCdE = 2.07944154$

and $AHKE = 0.2231435$

$ABCdE$ (when $AE:BC::10:1$.) = 2.025850

Therefore the Logar. of 10^3

is to the Log. of 2_1

as 2.302585

to 0.693147

An

